AN INVESTIGATION OF CONVECTIVE HEAT TRANSFER IN A CLOSED SPACE

Yu. G. Volodin and G. N. Dul'nev

Inzhenerno-Fizicheskii Zhurnal, Vol. 9, No. 5, pp. 603–608, 1965

UDC 536.25

An empirical formula is obtained for the convection coefficient in a closed parallelepiped.

Consider the natural convection in a closed volume (Fig. 1a) in which one of the bounding surfaces (shaded) is held at the temperature t_h while all other surfaces are held at the temperature t_c, where $t_h > t_c$. We shall refer to the surfaces at t_h and t_c as the heater and the cooler, respectively.

Heat transfer in a closed space can be analyzed in a manner analogous to that used for infinite layers, with the additional condition that the process is three-dimensional. The parameters governing natural convection in a closed space are

$$\lambda,\ c_p,\ \eta,\ \rho,\ g\beta,\ \vartheta,\ h,\ l_1,\ l_2,\ \alpha_1,\ \alpha_2; \qquad (1)$$

where α_1 and α_2 denote the film coefficients between the fluid and the hot and cold surfaces, respectively. In this work we shall derive a formula for the overall heat transfer coefficient between the heater and the cooler,

$$k = Q/\vartheta S. \qquad (2)$$

We introduce the notion of an equivalent thermal conductivity. We assume that the volume is filled with a solid material with conductivity λ_{eq} and that the total conductive heat flux through the solid is equal to the total convective heat flux through the fluid. Let the overall heat transfer coefficient and the equivalent thermal conductivity be related by

$$k = \lambda_{eq}/h. \qquad (3)$$

Taking account of (2) and (3), we can rewrite the system of parameters governing the problem in the form

$$\lambda_{eq},\ \lambda,\ c_p,\ \eta,\ \rho,\ g\beta,\ \vartheta,\ h,\ l_1,\ l_2. \qquad (4)$$

Experience shows that within a degree of accuracy sufficient for all practical purposes the two parameters l_1, l_2 can be replaced by a single equivalent dimension l, which we shall define presently. Thus it is required to find the functional relation between the equivalent conductivity λ_{eq} and all other parameters of (4),

$$\lambda_{eq} = f(\lambda,\ c_p,\ \eta,\ \rho,\ g\beta,\ \vartheta,\ h,\ l). \qquad (5)$$

Applying to (5) dimensional analysis, we obtain [3].

$$\varepsilon_k = \Psi(\mathrm{Gr},\ \mathrm{Pr},\ Z). \qquad (6)$$

Let us assume that the functional relation (6) is of the form

$$\varepsilon_k = D_0 Z_0^m (\mathrm{GrPr})^n. \qquad (7)$$

Taking into account known results regarding convection in infinite layers, we can assume that the process of convection in a closed space will depend on the orientation of the heated surface with respect to the volume. Following D. M. Boyarintsev [1], we introduce the coefficient L, defined as the ratio of the path followed by the convective current along the heated surface until it meets the cold surface to the vertical height of that path. Accordingly, L = 1 or 3 for vertical or horizontal heaters, respectively. *

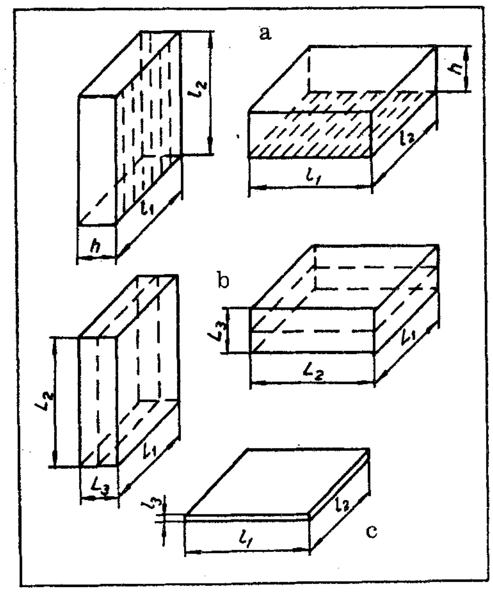

Fig. 1. Schematic picture of the bounded volumes (a), coolers (b), and heaters (c).

To take account of the orientation, we introduce this coefficient into (7), to obtain

$$\varepsilon_k = D_0 Z_0^m (\mathrm{GrPr}\,L)^n. \qquad (8)$$

We now represent the product $D_0 Z_0^m$ in the form

$$D_0 Z_0^m = D_0 (1 + h/l)^m, \qquad (9)$$

so that as l tends to infinity we will recover the known results for infinite layers [1,2,4–6]. Thus we represent the convection coefficient of the bounded space by the functional relation

$$\varepsilon_k = D_0 (1 + h/l)^m (\mathrm{GrPr}\,L)^n. \qquad (10)$$

*The ratio L = 3 follows from the assumption that the horizontal length of each convection cell in a horizontal layer is equal to twice the height of the layer [1]. (Translator.)

We shall determine the coefficients D_0, m, and n from experiments.

The experiments were carried out with the models shown in Fig. 1b. The coolers had the form of a parallelepiped and were made of 2 mm aluminum sheet. The heaters (Fig. 1c) were in the form of flat parallelepipeds and were so constructed that the temperature field was practically uniform. All surfaces of the heaters and coolers were painted with a paint having an emissivity of 0.92. The dimensions of the coolers (inner dimensions) and the heaters (outer dimensions) are given in the table. The heaters were fastened to the coolers in such a manner that the heat flux through their contact area was negligible with respect to the total flux.

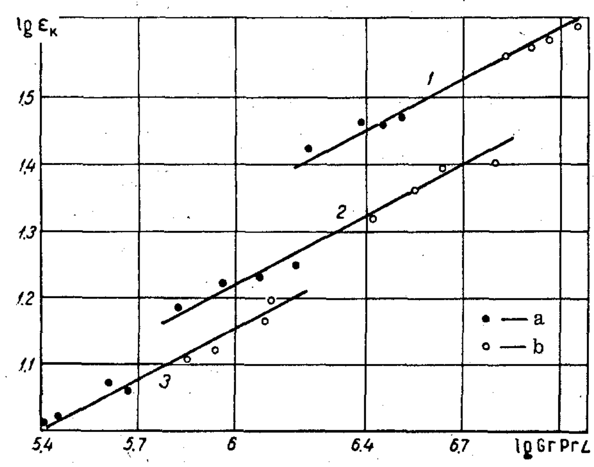

Fig. 2. Convection coefficients of bounded spaces. 1, 2, 3) Models No. 1, 2, and 3, respectively; a—vertical orientation; b—horizontal orientation.

When steady state was reached, the temperature of the heaters and the coolers was measured at 9 and 16 points by means of differential copper-constantan thermocouples (0.30 and 0.28 diameter wires). The readings of the thermocouples were used to calculate the surface-mean temperatures of the heater and the cooler. The heater was fed from a stabilized voltage source. The power dissipated by the heater was measured with a precision wattmeter.

In the vertical-heater models, the heat flux from the heater to the cooler is transmitted by convection and radiation in the vertical lateral spaces and by radiation and conduction through the air gap between the edges of the heater and the cooler. In the horizontal-heater models, the heat flux is transmitted by radiation from all surfaces, by convection in the upper horizontal space, and by conduction in the lower space and between the edges of the heater and the cooler. Therefore, to determine the convective heat flux, the radiative and conductive heat fluxes were subtracted in both cases from the total power dissipated by the heater.

The equivalent thermal conductivity was calculated from the formula

$$\lambda_{eq} = \frac{Qh}{(t_h - t_c)\, S}\,. \qquad (11)$$

The results were correlated in terms of dimensionless groups. The physical parameters in the groups were evaluated at the reference temperature:

$$t_m = 0.5\,(t_h + t_c). \qquad (12)$$

Figure 2 correlates the experimental results according to relation (10) in logarithmic coordinates. The slope corresponds to n = 0.25 and the intercepts on the ordinate axis represent the values of the product $D_0 Z_0^m$.

Choosing $l = \sqrt{l_1 l_2}$ as the equivalent dimensions of the heater, we can represent relation (9) in logarithmic coordinates (Fig. 3). This yields the values $D_0 = 0.24$ and $m = 4.15$. The empirical formula for the convection coefficient is then

$$\varepsilon_k = 0.24\,(1 + h/l)^{4.15}\,(\mathrm{Gr\,Pr}\,L)^{0.25} \qquad (13)$$

for parameters in the range

$$2.5\cdot 10^5 \leqslant \mathrm{Gr\,Pr}\,L \leqslant 1.2\cdot 10^7,$$
$$0.18 \leqslant h/l \leqslant 0.40.$$

This formula correlates the experimental data within 7%.

Due to the lack of published data on the convection coefficient in a parallelepiped, we could not compare

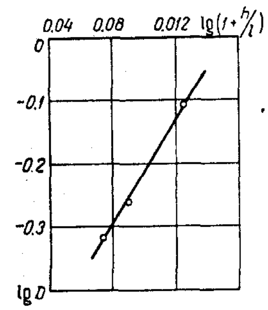

Fig. 3. Dependence of $D_0 Z_0^m$ on the ratio h/l.

our results with those of other authors. Therefore we shall use the following device: We assume that in (13) $h/l = 0$ (infinite layer) and we compare our results

Geometric Dimensions of the Models (in m)

No. of Model	L_1	L_2	L_3	l_1	l_2	l_3	h
1	0.339	0.339	0.130	0.337	0.337	0.013	0.061
2	0.296	0.572	0.200	0.295	0.570	0.013	0.094
3	0.339	0.339	0.240	0.337	0.337	0.013	0.114

with known data on the convection coefficient in infinite layers (Fig. 4). The results of this comparison allow us to conclude that the range of applicability of (13) can be extended to

$$0 \leqslant h/l \leqslant 0.40, \quad 10^4 \leqslant \mathrm{GrPr}\,L \leqslant 10^9.$$

Formula (13) can be rewritten in the form

$$\varepsilon_k = A_0 h (1 + h/l)^{4.15} \sqrt[4]{(t_h - t_c)/h}\, L^{0.25}, \qquad (14)$$

where

$$A_0 = 0.24\, (\beta\, g\, \mathrm{Pr})^{0.25}/\nu^{0.5}.$$

Here β is measured in $(^\circ\mathrm{C})^{-1}$, g in m · sec^{-2}; ν in $\mathrm{m}^2 \cdot \mathrm{sec}^{-1}$, h in m, and $L^{0.25} = 1$ for the vertical orientation and $L^{0.25} = 1.3$ for the horizontal orientation. The values of A_0 for air at $t_m = 0$, 50, 100, and 200° C are 26.6, 21.3, 18.2, and 14.0, respectively.

The overall heat transfer coefficient between the heater and the cooler is given by the formula

$$k = \left(1 + \frac{h}{l}\right)^{4.15} A \sqrt[4]{(t_h - t_c)/h}\, L^{0.25},$$

where $A = A_0 \lambda \ \mathrm{W \cdot m^{-7/4} \cdot C^{-5/4}}$. The values of A for air at $t_m = 0$, 50, 100, and 200° C, are 0.63, 0.58, 0.56, and 0.44, respectively.

NOTATION

λ) thermal conductivity; c_p) specific heat at constant pressure; η and ν) dynamic and kinematic viscosity; β) coefficient of thermal expansion; ϑ) temperature drop between heater and cooler; a) thermal diffusivity; ρ) density; Q) convective heat flux between heater and cooler: t_h and t_c) surface-mean temperatures of heater and cooler, respectively; S) area of heating surface of heater; $\varepsilon_k = \lambda_{eq}/\lambda$) dimensionless convection coefficient; $\mathrm{Gr} = g\beta\, h^3/\nu^2$; $\mathrm{Pr} = \nu/a$) Grashof and Prandtl numbers; $Z = h/l$; D_0, m and n) numerical coefficients.

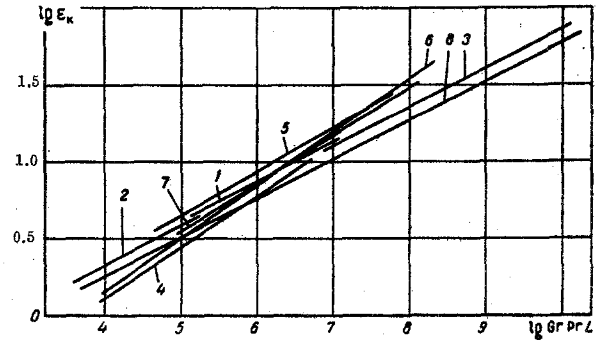

Fig. 4. Comparison of present results with published data on the convection coefficient of infinite layers. 1) Our data; 2) [5]; 3,4) [1]; 5,6) [6]; 7) [5]; 8) [4].

REFERENCES

1. D. M. Boyarintsev, ZhTF 20(9), 1084, 1950.
2. H. Gröber, S. Erk, and U. Grigull, Fundamentals of Heat Transfer [Russian translation], IL, 1958.
3. M. V. Kirpichev, Izv. AN SSSR, OTN, 9, 1330, 1953.
4. M. A. Mikheev, Fundamentals of Heat Transfer [in Russian], Gosenergoizdat, 1949.
5. W. Mull and H. Reiher, Beihefte z Gesundh—Ing. Reihe, no. 28, 1930.
6. H. Niemann, Gesundh.—Ing., 69, 224, 1948.

12 January 1965
Institute of Precision Mechanics and Optics, Leningrad